Exam 1 Algorithms 2012

Name ______________________

1. A. In a single sentence indicate what complexity is.
   B. Define Big O(g(n))
   C. Show that \(4n^2 + 8x +3\) is in \(O(n^2)\)
2. Define Big \(\Theta\) (g(n))
3. Is this true? Explain \(\Theta\)(g(n)) = \(O(g(n)) \cap \Omega(g(n))\)
4. Suppose that I would like to search an unsorted array \(A[0..n]\). I know that I could do a linear search but being cool I though a divide and conquer algorithm would be interesting. Here we divide the array in half and search both the upper half and lower half recursively. What is the complexity of this approach? Derive it. Is it better or worse than the linear method?
5. Solve this recurrence relation. \(T(n) = 2T(n/2) + n\). Give two algorithms that have this complexity.
6. Solve this recurrence \(T(n) = T(n/2) + 1\)
7. Solve this recurrence \(T(n) = 9T(n/3) + n\)
8. Suppose that I have an algorithm that has complexity \(\Theta(2n+10)\) and a different algorithm that has complexity \(\Theta((2n^2+3)\). If both algorithms solve the same problem which should I choose and over what range of \(n\) values. Draw a diagram and explain
9. How many binary digits does the base 10 number \(n\) get converted to?
10. How many hex digits does the base 10 number \(n\) get converted to.
11. Show that \(\log^2 n \in o(n)\)
12. Compare and contrast Merge Sort with Insertion sort with respect to time and space complexity. Worst, average and best complexity should be considered.
13. Give the recurrence relation that goes with the following function where \(A\) is an integer array \(A[0..n-1]\) and start and end represent the subscripts of the sub range of \(A\) that is processed
   Void Process ( int start, int end, int A[])
   {
       int p = (start + end)/3;
       if ( p>start){
           Process(start, p, A[]);
           Process (p+1,end, A[]);
           for (k=start;k<=end;k++); // missing ;
               for (j=start,j<=end);
           A[k]+=A[j]; // missing ;
       }.
   }
14. Write a C++ function that returns the sum of the digits contained in its unsigned int parameter. What is the complexity of this function.
15. Explain (draw diagram) how the second line is derived from the first

\[ A(n) = \frac{1}{n} \sum_{p=1}^{n} [A(p - 1) + A(n - p)] + n - 1 \]

\[ A(n) = \frac{2}{n} \sum_{p=1}^{n} A(p - 1) + n - 1 \]

16. Why is complexity important?

17. What is the complexity of this function? Explain.

```java
Inv(string s){
    int ct=0;
    for(int i=0;i<s.length();i++)
        for(int j=i+1;j<s.length();j++)
            if( s[i]>s[j] ) ct++;
    return 0;
}
```